

Theory for a Hanbury Brown Twiss experiment with a ballistically expanding cloud of cold atoms

J. Viana Gomes,^{1,2} A. Perrin,¹ M. Schellekens,¹

D. Boiron,^{1,*} C. I. Westbrook,¹ and M. Belsley²

¹*Laboratoire Charles Fabry de l'Institut d'Optique,*

UMR 8501 du CNRS et Université Paris 11, 91403 Orsay Cedex, France

²*Departamento de Fisica, Universidade do Minho,*

Campus de Gualtar, 4710-057 Braga, Portugal

Abstract

We have studied one-body and two-body correlation functions in a ballistically expanding, non-interacting atomic cloud in the presence of gravity. We find that the correlation functions are equivalent to those at thermal equilibrium in the trap with an appropriate rescaling of the coordinates. We derive simple expressions for the correlation lengths and give some physical interpretations. Finally a simple model to take into account finite detector resolution is discussed.

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*Electronic address: denis.boiron@institutoptique.fr; URL: <http://www.atomoptic.fr>

Whether a source emits photons or massive particles, if it is to be used in an interferometric experiment, an essential property is its coherence. The study of coherence in optics has shown that more than one kind of coherence can be defined [1]. The most familiar type of coherence is known as first order coherence and is related to the visibility of interference fringes in an interferometer. It is proportional to the value of the correlation function of the associated field. Second order coherence is less intuitive and corresponds to the correlation function of the intensity or squared modulus of the field. From a particle point of view, second order coherence is a way of quantifying density correlations and is related to the probability of finding one particle at a certain location given that another particle is present at some other location. Particle correlations can arise simply from exchange symmetry effects and exist even when there is no interaction between the particles. This fact was clearly demonstrated in the celebrated Hanbury Brown Twiss experiment which showed a second order correlation for photons coming from widely separated points in a thermal source such as a star [2].

Analogous correlations in massive particles have also been studied, particularly in the field of nuclear physics [3, 4, 5, 6, 7]. Spatial correlations using low energy electrons have also been studied [8, 9]. The advent of laser and evaporative cooling techniques has also made it possible to look for correlations between neutral atoms and recently a wide variety of different situations have been studied [10, 11, 12, 13, 14, 15, 16]. Correlation phenomena are generally richer when using massive particles because they can be either Bosons or Fermions, they often have a more complex internal structure and a large range of possible interactions with each other. In the field of ultra-cold atoms, the many theoretical papers to date have included treatments of bosons in a simple three dimensional harmonic trap [17, 18], a 1D bosonic cloud in the Thomas Fermi regime and Tonks-Girardeau limit [19, 20, 21], the Mott-insulator or superfluid phase for atoms trapped in optical lattices [22] and the 2D gas [23].

Almost all these theoretical treatments have dealt with atomic clouds at thermal equilibrium. On the other hand, all the experiments so far except Ref.[16] have measured correlations in clouds released from a trap which expand under the influence of gravity and possibly interatomic interactions. It is generally not trivial to know how the correlation properties evolve during expansion. Moreover, matter waves have different dispersion characteristics than light. All this raises interesting questions concerning the value of the

correlation lengths during the atomic cloud expansion. In particular we would like to know how to use the results of Ref.[17] to analyze the experimental results of Ref.[15], a conceptually simple experiment in which second order correlations were measured in a freely expanding cloud of metastable helium atoms. The correlation length was defined as the characteristic length of the normalized second order correlation function. We will use the same definition in this paper (see section I A for details).

To illustrate a more general question that comes up in thinking about the coherence of de Broglie waves, consider a beam of particles with mean velocity v hitting a detector. Two obvious length scales come immediately to mind, the de Broglie wavelength $\hbar/(m\Delta v)$ associated with the velocity spread Δv and the length associated with the inverse of the energy spread of the source $\hbar v/m(\Delta v)^2$. These two scales are obviously very different if v is large compared to the velocity spread. In this paper, we will show that in an experiment such as [15], the correlation length corresponds to neither of the above length scales, although they can be relevant in other situations. We find that the correlation length after an expansion time t of a cloud of initial size s is $\hbar t/ms$. This result is the atom optical analog of the van Cittert-Zernike theorem [24]. It has also been stated in a different form in Ref. [25]. For the special case of an ideal gas in a harmonic trap of oscillation frequency ω , the correlation length can be recast as $\lambda\omega t$ where λ is the thermal de Broglie wavelength. Hence the correlation length after expansion is simply dilated compared to that at equilibrium with the same scaling factor as the spatial extent of the cloud itself.

We will confine ourselves here to the case of a cloud of non interacting atoms released suddenly from a harmonic trap. The paper is organized as follows. We will begin in section I with some simple definitions and general results about the correlation properties of a non-interacting cloud both at thermal equilibrium in a trapping potential and after a ballistic expansion. Without making any assumptions about the form of the trapping potential, we can only find simple analytical results in the limit of a non-degenerate gas. Next we will make a more exact and careful treatment by specializing to the very important case of a harmonic potential. We introduce the flux operator [26] involved in the experimental electronic detection with metastable helium and then calculate the correlation function of the flux. We will summarize the results and give a physical interpretation in section III. This interpretation will allow us to comment on the rather different case of a continuous beam as in the experiments of Ref.[7, 10, 14]. In section IV we will use our results to analyse

the experimentally important problem of finite detector resolution. Finally, the appendix adds some detailed calculations concerning the expressions found in section II B.

I. GENERAL RESULTS ON CORRELATION FUNCTIONS OF NON-INTERACTING GASES

Here we recall some basic results concerning the density and first and second order correlation functions for a cloud of non-interacting bosons at thermal equilibrium. A more detailed analysis can be found in Ref.[17]. Theoretical treatments that take into account interatomic interactions can be found in Ref.[17, 18, 27]. We also give some approximate results for a non-interacting gas after it has expanded from a trap.

A. Definitions

Consider a cloud of N atoms at thermal equilibrium at a temperature T , confined in a trapping potential. This potential is characterized by $\{\epsilon_{\mathbf{j}}, \psi_{\mathbf{j}}^0(\mathbf{r})\}$ the energy and wavefunction of level \mathbf{j} (here supposed non-degenerate for simplicity). In second quantization, one defines the field operators

$$\hat{\Psi}^\dagger(\mathbf{r}) = \sum_{\mathbf{j}} \psi_{\mathbf{j}}^*(\mathbf{r}) \hat{a}_{\mathbf{j}}^\dagger, \quad \hat{\Psi}(\mathbf{r}) = \sum_{\mathbf{j}} \psi_{\mathbf{j}}(\mathbf{r}) \hat{a}_{\mathbf{j}}.$$

The operator $\hat{a}_{\mathbf{j}}^\dagger$ creates and $\hat{a}_{\mathbf{j}}$ annihilates one particle in state $|\psi_{\mathbf{j}}\rangle$ whereas $\hat{\Psi}^\dagger(\mathbf{r})$ creates and $\hat{\Psi}(\mathbf{r})$ annihilates a particle at position \mathbf{r} .

Correlation functions and the atomic density are statistical averages of such field operators. We use the Bose-Einstein distribution, $\langle \hat{a}_{\mathbf{j}}^\dagger \hat{a}_{\mathbf{k}} \rangle = \delta_{\mathbf{j} \mathbf{k}} (e^{\beta(\epsilon_{\mathbf{j}} - \mu)} - 1)^{-1}$ where $\beta = 1/(k_B T)$, k_B is the Boltzmann constant and μ is the chemical potential. The value of μ ensures the normalization $\sum_{\mathbf{j}} \langle \hat{a}_{\mathbf{j}}^\dagger \hat{a}_{\mathbf{j}} \rangle = N$. We can then define

- the first order correlation function $G^{(1)}(\mathbf{r}, \mathbf{r}') = \langle \hat{\Psi}^\dagger(\mathbf{r}) \hat{\Psi}(\mathbf{r}') \rangle$,
- the second order correlation function $G^{(2)}(\mathbf{r}, \mathbf{r}') = \langle \hat{\Psi}^\dagger(\mathbf{r}) \hat{\Psi}(\mathbf{r}) \hat{\Psi}^\dagger(\mathbf{r}') \hat{\Psi}(\mathbf{r}') \rangle$
- and the density $\rho_{eq}(\mathbf{r}) = \langle \hat{\Psi}^\dagger(\mathbf{r}) \hat{\Psi}(\mathbf{r}) \rangle = G^{(1)}(\mathbf{r}, \mathbf{r})$.

Several other first and second order correlation functions can be defined (see below) but these are the most common ones. The first order correlation function appears in interference experiments whereas second order correlation functions are related to intensity interference or density fluctuation. First and second-order correlation functions are connected for thermal non-interacting atomic clouds. The $G^{(2)}$ function contains a statistical average of the type $\langle \hat{a}_j^\dagger \hat{a}_k \hat{a}_l^\dagger \hat{a}_n \rangle$ which can be calculated through the thermal averaging procedure (Wick theorem [28]). One finds $\langle \hat{a}_j^\dagger \hat{a}_k \hat{a}_l^\dagger \hat{a}_n \rangle = \langle \hat{a}_j^\dagger \hat{a}_j \rangle \langle \hat{a}_k^\dagger \hat{a}_k \rangle (\delta_{jl} \delta_{kn} + \delta_{jn} \delta_{kl}) + \langle \hat{a}_j^\dagger \hat{a}_j \rangle \delta_{kl} \delta_{jn}$, which leads to

$$G^{(2)}(\mathbf{r}, \mathbf{r}') = \rho_{eq}(\mathbf{r}) \rho_{eq}(\mathbf{r}') + |G^{(1)}(\mathbf{r}, \mathbf{r}')|^2 + \rho_{eq}(\mathbf{r}) \delta(\mathbf{r} - \mathbf{r}')$$

The last term is the so-called shot-noise term. It will be neglected in the following because it is proportional to N whereas the others are proportional to N^2 .

It is convenient to define a normalized second order correlation function

$$g^{(2)}(\mathbf{r}, \mathbf{r}') = \frac{G^{(2)}(\mathbf{r}, \mathbf{r}')}{\rho_{eq}(\mathbf{r}) \rho_{eq}(\mathbf{r}')}.$$

If the cloud has a finite correlation length, then for distances larger than this length the first-order correlation function vanishes. Then $g^{(2)}(\mathbf{r}, \mathbf{r}) = 2$ and $g^{(2)}(\mathbf{r}, \mathbf{r}') \rightarrow 1$ when $|\mathbf{r} - \mathbf{r}'| \rightarrow \infty$. This means that the probability of finding two particles close to each other is enhanced by a factor of 2, compared to the situation where they are far apart. This is the famous bunching effect first observed by Hanbury Brown and Twiss with light [2].

The above expression of the $G^{(2)}$ function cannot be applied in the vicinity and below the Bose-Einstein transition temperature. The calculation of $\langle \hat{a}_j^\dagger \hat{a}_k \hat{a}_l^\dagger \hat{a}_n \rangle$ is performed in the grand canonical ensemble which assumes the existence of a particle reservoir that does not exist for the condensate. It is well known [29] that this gives unphysically large fluctuations of the condensate at low enough temperature. This pathology disappears at the thermodynamic limit if there is an interatomic interaction [29]. It has also been shown that it cancels for a finite number of non-interacting particles if one uses the more realistic canonical ensemble [30]. One way to keep using the grand canonical ensemble is to add the canonical result for the ground-state [17]. This approach is validated by the results in Ref.[30] and will be used in the following. The largest deviation is expected to occur near the transition temperature [30]. The contribution of the ground state is $-\langle \hat{a}_0^\dagger \hat{a}_0 \rangle^2 \delta_{j0} \delta_{k0} \delta_{l0} \delta_{n0}$. Then, with ρ_0 the ground-state density, it follows that,

$$G^{(2)}(\mathbf{r}, \mathbf{r}') = \rho_{eq}(\mathbf{r})\rho_{eq}(\mathbf{r}') + |G^{(1)}(\mathbf{r}, \mathbf{r}')|^2 - \rho_0(\mathbf{r})\rho_0(\mathbf{r}') \quad (1)$$

The normalized second order then becomes

$$g^{(2)}(\mathbf{r}, \mathbf{r}') = 1 + \frac{|G^{(1)}(\mathbf{r}, \mathbf{r}')|^2}{\rho_{eq}(\mathbf{r})\rho_{eq}(\mathbf{r}')} - \frac{\rho_0(\mathbf{r})\rho_0(\mathbf{r}')}{\rho_{eq}(\mathbf{r})\rho_{eq}(\mathbf{r}')}$$

Because the ground state density is negligible for a thermal cloud, the normalized correlation function $g^{(2)}(\mathbf{r}, \mathbf{r}')$ still goes from 2 to 1 as the separation of \mathbf{r} and \mathbf{r}' increases. On the other hand, for a BEC at $T = 0$, only the ground-state is occupied. Then $|G^{(1)}(\mathbf{r}, \mathbf{r}')|^2 = \rho_{eq}(\mathbf{r})\rho_{eq}(\mathbf{r}') = \rho_0(\mathbf{r})\rho_0(\mathbf{r}')$ and $g^{(2)}(\mathbf{r}, \mathbf{r}') = 1$. The amount of particle bunching present in the second order correlation function can be quantified as $g^{(2)}(\mathbf{r}, \mathbf{r}') - 1$ and this typically decays exponentially as the modulus squared of the separation between the two points increases. We define the correlation length to be the characteristic length over which the amount of particle bunching decays, that is the distance over which $g^{(2)}(\mathbf{r}, \mathbf{r}') - 1$ decays to $1/e$ of its maximum value. The correlation length of a BEC is infinite. Such a system is said to exhibit bunching at high temperature over the correlation length and no bunching in the condensed phase.

B. Correlations in an expanding cloud

In most experiments, particle correlations and other characteristics are not directly measured in the atom cloud, (Ref. [16] is an exception). Rather, the cloud is released from a trap and allowed to expand during a “time of flight” before detection. For a sufficiently long time of flight, and neglecting interactions between the atoms, the positions one measures at a detector reflect the initial momenta of the particles. The results of section I A concerning the correlation functions in position space all have analogs in momentum space. In fact the correlation functions in the two reciprocal spaces are closely related. At equilibrium, i.e. inside the trap, the following relationships can be easily derived:

$$\begin{aligned} \int d\mathbf{p} G^{(1)}(\mathbf{p}, \mathbf{p}) e^{-i\mathbf{p}\cdot\mathbf{r}/\hbar} &= \int d\mathbf{R} G^{(1)}(\mathbf{R} - \mathbf{r}/2, \mathbf{R} + \mathbf{r}/2) \\ \int d\mathbf{r} G^{(1)}(\mathbf{r}, \mathbf{r}) e^{i\mathbf{q}\cdot\mathbf{r}/\hbar} &= \int d\mathbf{P} G^{(1)}(\mathbf{P} - \mathbf{q}/2, \mathbf{P} + \mathbf{q}/2) \end{aligned}$$

In other words, the spatial correlation length is related to the width of the momentum distribution and the momentum correlation length is related to the width of the spatial

distribution *i.e.* the size of the cloud. No equally simple and general relationship holds for the second order correlation functions. This is because, close to the BEC transition temperature, and at points where the ground state wave function is not negligible, the special contribution of the ground state, the last term in Eq. 1 must be included, and this contribution depends on the details of the confining potential. On the other hand, for an ideal gas far from the transition temperature one can neglect the ground state density, make the approximation that the correlation length is very short, neglect commutators such as $[\hat{\mathbf{r}}, \hat{\mathbf{p}}]$, and then write the thermal density operator as $\hat{\sigma} = e^{-\beta \frac{\hat{\mathbf{p}}^2}{2m}} e^{-\beta V(\hat{\mathbf{r}})}$. These approximations lead to:

$$G^{(2)}(\mathbf{p}, \mathbf{p}') = \rho_{eq}(\mathbf{p})\rho_{eq}(\mathbf{p}') + |G^{(1)}(\mathbf{p}, \mathbf{p}')|^2$$

and,

$$G^{(1)}(\mathbf{P} - \mathbf{q}/2, \mathbf{P} + \mathbf{q}/2) \sim e^{-\beta \frac{\mathbf{P}^2}{2m}} \int d\mathbf{r} e^{-\beta V(\mathbf{r})} e^{i\frac{\mathbf{q}\cdot\mathbf{r}}{\hbar}}$$

One sees that in this limit, the interesting part of $G^{(2)}$ in momentum space is proportional to the square of the Fourier transform of the density distribution and independent of the mean momentum \mathbf{P} . This result is the analog of the van Cittert-Zernike theorem [24]. For a trapped cloud of size s_α in the α direction, one has a momentum correlation “length” given by:

$$p_\alpha^{(coh)} = \frac{\hbar}{s_\alpha}. \quad (2)$$

If atoms are suddenly released from a trap and allowed to freely evolve for a sufficiently long time t , the positions of the particles reflect their initial momenta and the spatial correlation length at a detector is given by

$$l_\alpha^{(d)} = \frac{p_\alpha^{(coh)}}{m} t = \frac{\hbar t}{m s_\alpha} \quad (3)$$

The normalized second order correlation function is then a Gaussian of rms width $l^{(d)}/\sqrt{2}$. This result was experimentally confirmed in Ref. [15]. One wonders however, to what extent the approximations we have made are valid. The clouds used in Ref. [15] were in fact very close to the transition temperature so that effects due to the Bose nature of the density matrix may be important. Although the time of flight was very long, it is useful to quantify the extent to which identifying the momentum correlation length in the trap with the spatial correlation length at the detector is accurate. Finally, the effect of gravity on the falling atoms never appears in the above approximate treatment, and we would like to clarify the

role it plays. In order to answer these questions we undertake a more careful calculation. We will confine ourselves to atoms initially confined in a harmonic trap, a good approximation to the potential used in most experiments, and happily, one for which the eigenstates and energies are known exactly.

II. DENSITY AND CORRELATION FUNCTIONS FOR A HARMONIC TRAP

A. At equilibrium in the trap

The eigenfunctions for a 3-dimensional harmonic potential of oscillation frequency ω_α in the α direction, are given by:

$$\psi_{\mathbf{j}}^0(\mathbf{r}) = \prod_{\alpha=x,y,z} A_{j_\alpha} e^{-\frac{r_\alpha^2}{2\sigma_\alpha^2}} H_{j_\alpha}(r_\alpha/\sigma_\alpha).$$

Here $\sigma_\alpha = \sqrt{\frac{\hbar}{m\omega_\alpha}}$ is the harmonic oscillator ground-state size, H_{j_α} is the Hermite polynomial of order j_α and $A_{j_\alpha} = (\sqrt{\pi}\sigma_\alpha 2^{j_\alpha}(j_\alpha!)^{-1/2})$. The eigenenergies are given by $\epsilon_{\mathbf{j}} = \sum_{\alpha=x,y,z} \hbar\omega_\alpha(j_\alpha + 1/2)$. Then [17, 29], with $\tau_\alpha = \beta\hbar\omega_\alpha$ and $\tilde{\mu} = \mu - \hbar \sum \omega_\alpha/2$, one finds:

$$\rho_{eq}(\mathbf{r}) = \frac{1}{\pi^{3/2}} \sum_{l=1}^{\infty} e^{\beta l \tilde{\mu}} \prod_{\alpha} \frac{1}{\sigma_\alpha \sqrt{1 - e^{-2\tau_\alpha l}}} e^{-\tanh(\frac{\tau_\alpha l}{2}) \frac{r_\alpha^2}{\sigma_\alpha^2}}$$

and

$$G^{(1)}(\mathbf{r}, \mathbf{r}') = \frac{1}{\pi^{3/2}} \sum_{l=1}^{\infty} e^{\beta l \tilde{\mu}} \prod_{\alpha} \frac{1}{\sigma_\alpha \sqrt{1 - e^{-2\tau_\alpha l}}} \exp \left[-\tanh\left(\frac{\tau_\alpha l}{2}\right) \left(\frac{r_\alpha + r'_\alpha}{2\sigma_\alpha}\right)^2 - \coth\left(\frac{\tau_\alpha l}{2}\right) \left(\frac{r_\alpha - r'_\alpha}{2\sigma_\alpha}\right)^2 \right].$$

The above expressions can be transformed into more familiar forms in limiting cases:

- For high temperature, $\mu \rightarrow -\infty$ and one recovers the Maxwell-Boltzmann distribution. The density is $\rho_{eq}(\mathbf{r}) = \frac{N}{\lambda^3} \prod_{\alpha} \tau_\alpha e^{-\frac{\tau_\alpha}{2} \frac{r_\alpha^2}{\sigma_\alpha^2}}$ with $\lambda = \frac{\hbar\sqrt{2\pi}}{\sqrt{mk_B T}}$ the thermal de Broglie wavelength. The size of the cloud is $s_\alpha = \sigma_\alpha / \sqrt{\tau_\alpha} = \sqrt{\frac{k_B T}{m\omega_\alpha^2}}$.

The first order correlation function is

$$G^{(1)}(\mathbf{r}, \mathbf{r}') = \frac{N}{\lambda^3} \prod_{\alpha} \tau_\alpha e^{-\frac{\tau_\alpha}{2} \left(\frac{r_\alpha + r'_\alpha}{2\sigma_\alpha}\right)^2} e^{-\pi \left(\frac{r_\alpha - r'_\alpha}{\lambda}\right)^2}. \quad (4)$$

Using our definition, the correlation length is $l^{(t)} = \lambda/\sqrt{2\pi}$.

- For a temperature close to but above the Bose-Einstein transition temperature, one has to keep the summation over the index l . The density is $\rho_{eq}(\mathbf{r}) = \frac{1}{\lambda^3} g_{3/2}[e^{\beta\tilde{\mu}} \prod_{\alpha} e^{-\frac{\tau_{\alpha}}{2} \frac{r_{\alpha}^2}{\sigma_{\alpha}^2}}]$ where $g_a(x) = \sum_{l=1}^{\infty} x^l / l^a$ is a Bose function. The first order correlation function is

$$G^{(1)}(\mathbf{r}, \mathbf{r}') = \frac{1}{\lambda^3} \sum_{l=1}^{\infty} \frac{e^{l\beta\tilde{\mu}}}{l^{3/2}} \prod_{\alpha} e^{-\frac{\tau_{\alpha} l}{2} (\frac{\mathbf{r}_{\alpha} + \mathbf{r}'_{\alpha}}{2\sigma_{\alpha}})^2} e^{-\frac{\pi}{l} (\frac{\mathbf{r}_{\alpha} - \mathbf{r}'_{\alpha}}{\lambda})^2}.$$

As the temperature decreases, the number of values of l that contribute significantly to the sum increases. It is then clear from the above expression for $G^{(1)}$ that the correlation length near the center of the trap will increase and that the normalized correlation function is no longer Gaussian. Far from the center, only the $l = 1$ term is important and the correlation function remains Gaussian. Thus close to degeneracy the correlation length is position-dependent (for an explicit example see Sec.II B 5).

- Near and below the transition temperature, the second order correlation function is given by Eq. (1) with $\rho_0(\mathbf{r}) = \frac{e^{\beta\tilde{\mu}}}{1 - e^{\beta\tilde{\mu}}} \prod_{\alpha} \frac{e^{-\tau_{\alpha}^2 / \sigma_{\alpha}^2}}{(\sqrt{\pi}\sigma_{\alpha})^3}$. As the temperature decreases, the correlation at zero distance, $g^{(2)}(0, 0)$ decreases from 2 to 1 and the correlation length increases. Around the transition temperature, $g^{(2)}(0, 0)$ is already significantly different from 2 since the condensate peak density is already very large for a non-interacting harmonically trapped cloud [31]. At $T = 0$, the correlation length is infinite and $g^{(2)}(\mathbf{r}, \mathbf{r}') = 1$.

B. Correlations in a harmonically trapped cloud after expansion

Here we consider the cloud after expansion. First we discuss two classes of detection methods which must be distinguished before calculating correlation functions.

1. Detection

We assume that the trapping potential is switched off instantaneously at $t = 0$. The cloud expands and falls due to gravity. Two types of detection can be performed:

- *Snap shot.* An image is taken of the entire cloud at $t = t_0$. We have then access to

$$G_{im.}^{(2)}(\mathbf{r}, t_0; \mathbf{r}', t_0) = \langle \hat{\Psi}^{\dagger}(\mathbf{r}, t_0) \hat{\Psi}(\mathbf{r}, t_0) \hat{\Psi}^{\dagger}(\mathbf{r}', t_0) \hat{\Psi}(\mathbf{r}', t_0) \rangle$$

The usual imaging technique is absorption, and so one has access to the above correlation functions integrated along the imaging beam axis. This was used for the experiments of Ref.[12, 13].

- *Flux measurement.* The atoms are detected when they cross a given plane. We will only consider the situation in which this plane is horizontal at $z = H$. One has access to

$$G_{fl.}^{(2)}(\mathbf{r} = \{x, y, z = H\}, t; \mathbf{r}' = \{x', y', z' = H\}, t') = \langle \hat{I}(\mathbf{r}, t) \hat{I}(\mathbf{r}', t') \rangle$$

where \hat{I} is the flux operator defined below. The detection systems required for such experiments correspond most closely to those of Refs. [10, 15], in which a micro-channel plate, situated below the trapped cloud, recorded the arrival times and in one case the positions of the atoms. It also corresponds closely to imaging a cloud that crosses a thin sheet of light [32], or to the experiment of Ref.[14], in which the transmission of a high finesse optical cavity records atoms as they cross the beam.

These two correlation functions are different, but if the detection is performed after a long time of flight, they are in fact nearly equivalent. This equivalence will be discussed in the following.

The flux operator is defined quantum-mechanically by

$$\hat{I}(\mathbf{r}, t) = \frac{\hbar}{m} \text{Im} [\hat{\Psi}^\dagger(\mathbf{r}, t) \partial_z \hat{\Psi}(\mathbf{r}, t)] = \frac{\hbar}{2mi} [\hat{\Psi}^\dagger(\mathbf{r}, t) \partial_z \hat{\Psi}(\mathbf{r}, t) - \partial_z \hat{\Psi}^\dagger(\mathbf{r}, t) \hat{\Psi}(\mathbf{r}, t)]$$

The flux has thus the dimensions of a density times a velocity. We will give the explicit expression of this velocity in the section II B 4. Here, the atomic field operators $\hat{\Psi}(\mathbf{r}, t)$ depend on space coordinates as well as on time. They represent the time evolution of the atomic field during the flight of the atoms, falling from the trap. The field operators for the falling cloud can be easily derived if we assume that there are no interactions between the atoms and that the occupation number in each mode is constant (as in free expansion). In this case, these operators can be defined as

$$\hat{\Psi}^\dagger(\mathbf{r}, t) = \sum_{\mathbf{j}} \psi_{\mathbf{j}}^*(\mathbf{r}, t) \hat{a}_{\mathbf{j}}^\dagger, \quad \hat{\Psi}(\mathbf{r}, t) = \sum_{\mathbf{j}} \psi_{\mathbf{j}}(\mathbf{r}, t) \hat{a}_{\mathbf{j}}$$

where the spatiotemporal dependence is carried by the wave function and the statistical occupation by the creation and annihilation operators.

2. Ballistic expansion of a harmonic oscillator stationary state

After switching off the trap, the harmonic oscillator wave-functions noted $\psi_{\mathbf{j}}^0$ are no longer stationary states. There are two ways to calculate the correlation after expansion: propagation of wavefunctions or propagation of the density matrix (the Schrödinger or the Heisenberg picture). In the following we will use the first approach which is physically more transparent (see [33] for the Heisenberg picture).

The ballistic expansion of a cloud is easy to calculate with the appropriate Green function. The Green function K is defined as

$$\psi_{\mathbf{j}}(\mathbf{r}, t) = \int_{-\infty}^{\infty} d\mathbf{r}_0 K(\mathbf{r}, t; \mathbf{r}_0, t_0) \psi_{\mathbf{j}}^0(\mathbf{r}_0, t_0).$$

As the $\psi_{\mathbf{j}}^0$ functions are stationary states for $t < 0$, we can take $t_0 = 0$ in the following. The Green function for particles in an arbitrarily time-varying quadratic potential is known [34]. After expansion, the potential is only due to gravity and the Green function is then

$$K(\mathbf{r}, t; \mathbf{r}_0) = \left(\frac{m}{2i\pi\hbar t} \right)^{3/2} e^{ia(\mathbf{r}-\mathbf{r}_0)^2} e^{ib(z+z_0)} e^{-ic}$$

with $a = \frac{m}{2\hbar t}$, $b = \frac{mgt}{2\hbar}$ and $c = \frac{mg^2 t^3}{24\hbar}$.

One can then derive an analytical expression of $\psi_{\mathbf{j}}(\mathbf{r}, t)$ [35, 36]:

$$\psi_{\mathbf{j}}(\mathbf{r}, t) = e^{i\phi(\mathbf{r}, t)} \prod_{\alpha} \frac{e^{ij_{\alpha}(\delta_{\alpha} + 3\pi/2)}}{\sqrt{\omega_{\alpha} t - i}} \psi_{\mathbf{j}}^0(\tilde{\mathbf{r}}) \quad (5)$$

where $\delta_{\alpha} = \tan^{-1}[\frac{1}{\omega_{\alpha} t}]$,

$$\phi(\mathbf{r}, t) = \frac{m}{2\hbar t} \left[(\tilde{x}\omega_x t)^2 + (\tilde{y}\omega_y t)^2 + (\tilde{z}\omega_z t)^2 + 2gt^2(z - \frac{1}{8}gt^2) \right] - c - \frac{3\pi}{4} \quad (6)$$

and, with $\tilde{\mathbf{r}} = \{\tilde{x}, \tilde{y}, \tilde{z}\}$,

$$\tilde{x} = \frac{x}{\sqrt{1 + \omega_x^2 t^2}}, \tilde{y} = \frac{y}{\sqrt{1 + \omega_y^2 t^2}}, \tilde{z} = \frac{H - \frac{1}{2}gt^2}{\sqrt{1 + \omega_z^2 t^2}} \quad (7)$$

In the case of flux measurement, the position of the detector is fixed at $z = H$. The phase $\phi(\tilde{x}, \tilde{y}, t)$ is global as it does not depend on the index \mathbf{j} ; it will cancel in second order correlation measurements. This is in contrast to interferometric measurements where it is this phase that gives rise to fringes. The above results show that after release, the wavefunction is identical to that in the trap except for a phase factor and a scaling factor in the positions [37]. This scaling is obviously a property of a harmonic potential, and it considerably simplifies the expression of the correlation functions as we will see below.

3. Flux operator

Using $\partial_z H_n(z) = 2nH_{n-1}(z)$, the spatial derivative of the wavefunction can be written:

$$\partial_z \psi_{\mathbf{j}}(\mathbf{r}, t) = \frac{m}{\hbar} \left\{ [iv_2 - v_1] \psi_{j_z}(z, t) - iv_3 \sqrt{j_z} \psi_{j_z-1}(z, t) \right\} \psi_{j_x}(x, t) \psi_{j_y}(y, t)$$

where the velocities v_1 , v_2 and v_3 are time dependent and are given by

$$v_1(t) = \omega_z \frac{H - \frac{1}{2}gt^2}{1 + \omega_z^2 t^2} \quad (8)$$

$$v_2(t) = \frac{1}{t} \left[H + \frac{1}{2}gt^2 - \frac{H - \frac{1}{2}gt^2}{1 + \omega_z^2 t^2} \right] \quad (9)$$

$$v_3(t) = \frac{\sqrt{2}\omega_z \sigma_z}{\sqrt{1 + \omega_z^2 t^2}} e^{i\delta_z} \quad (10)$$

The velocity v_2 is usually much larger than the other two and will give the dominant contribution for the mean flux and the second order correlation function. An atom with zero initial velocity will acquire after a time t a velocity gt which is close to $v_2(t)$. The flux operator is,

$$\hat{I}(\mathbf{r}, t) = \sum_{\mathbf{j}, \mathbf{k}} \left[v_2 \psi_{\mathbf{j}}^* \psi_{\mathbf{k}} - \frac{1}{2} \left(v_3 \sqrt{k} \psi_{\mathbf{j}}^* \psi_{\mathbf{k}-\mathbf{1}_z} + v_3^* \sqrt{j} \psi_{\mathbf{j}-\mathbf{1}_z}^* \psi_{\mathbf{k}} \right) \right] \hat{a}_{\mathbf{j}}^\dagger \hat{a}_{\mathbf{k}} \quad (11)$$

where $\mathbf{j} - \mathbf{1}_z$ is the vector $(j_x, j_y, j_z - 1)$ and where we write $\psi = \psi(\mathbf{r}, t)$.

4. Mean density and mean flux

We will first calculate the mean density $\rho(\mathbf{r}, t) = \langle \hat{\Psi}^\dagger(\mathbf{r}, t) \hat{\Psi}(\mathbf{r}, t) \rangle$. Using Eq.(5), one finds easily that $\rho(\mathbf{r}, t) = \frac{1}{\prod_{\alpha} \sqrt{1 + \omega_{\alpha}^2 t^2}} \rho_{eq}(\tilde{\mathbf{r}})$. This means that the density has the same form during expansion up to an anisotropic scale factor given by Eq.(7) [37, 38]. The statistical average of Eq.(11) leads to

$$\langle \hat{I}(\mathbf{r}, t) \rangle = \sum_{\mathbf{j}} \left[v_2 |\psi_{\mathbf{j}}|^2 - \frac{\sqrt{j_z}}{2} \left(v_3 \psi_{j_z}^* \psi_{j_z-1} + v_3^* \psi_{j_z} \psi_{j_z-1}^* \right) |\psi_{j_x} \psi_{j_y}|^2 \right] \langle \hat{a}_{\mathbf{j}}^\dagger \hat{a}_{\mathbf{j}} \rangle.$$

Because $v_3 \psi_{j_z}^* \psi_{j_z-1} = i \frac{|v_3|}{\sqrt{1 + \omega_z^2 t^2}} \psi_{j_z}^0(\tilde{z}) \psi_{j_z-1}^0(\tilde{z}) = -v_3^* \psi_{j_z} \psi_{j_z-1}^*$, the second term cancels out. Then, without any approximation,

$$\langle \hat{I}(\mathbf{r}, t) \rangle = \frac{v_2(t)}{\prod_{\alpha} \sqrt{1 + \omega_{\alpha}^2 t^2}} \rho_{eq}(\tilde{\mathbf{r}}) = v_2(t) \rho(\mathbf{r}, t)$$

The flux is proportional to the density of a cloud at thermal equilibrium with rescaled coordinates. This means that the mean flux of an expanding non-interacting cloud is proportional to the atomic density without any approximation. This results holds with and without gravity taken into account.

5. Second order correlation

Here we calculate the correlation functions. A discussion is given in the next section. The snap-shot correlation function is

$$G_{im.}^{(2)}(\mathbf{r}, t; \mathbf{r}', t) = \sum_{\mathbf{j}, \mathbf{k}, \mathbf{l}, \mathbf{n}} \psi_{\mathbf{j}}^* \psi_{\mathbf{k}} \times \psi_{\mathbf{l}}'^* \psi_{\mathbf{n}}' \langle \hat{a}_{\mathbf{j}}^\dagger \hat{a}_{\mathbf{k}} \hat{a}_{\mathbf{l}}^\dagger \hat{a}_{\mathbf{n}} \rangle.$$

Using Eq.(5), one finds, without any approximation (except the neglect of the shot-noise term):

$$G_{im}^{(2)}(\mathbf{r}, t; \mathbf{r}', t) = \frac{1}{\prod_{\alpha} (1 + \omega_{\alpha}^2 t^2)} \left(\rho_{eq}(\tilde{\mathbf{r}}) \rho_{eq}(\tilde{\mathbf{r}}') + |G^{(1)}(\tilde{\mathbf{r}}, \tilde{\mathbf{r}}')|^2 - \rho_0(\tilde{\mathbf{r}}) \rho_0(\tilde{\mathbf{r}}') \right).$$

As in the case of the mean density, the snap-shot correlation function has the same form as in the trap except for an anisotropic scale factor.

The calculation of $G_{fl}^{(2)}$ is similar:

$$\langle \hat{I}(\mathbf{r}, t) \hat{I}(\mathbf{r}', t') \rangle = - \left(\frac{\hbar}{2m} \right)^2 \sum_{\mathbf{j}, \mathbf{k}, \mathbf{l}, \mathbf{n}} [\psi_{\mathbf{j}}^* (\partial_z \psi_{\mathbf{k}}) - (\partial_z \psi_{\mathbf{j}}^*) \psi_{\mathbf{k}}] \times [\psi_{\mathbf{l}}'^* (\partial_z \psi_{\mathbf{n}}') - (\partial_z \psi_{\mathbf{l}}'^*) \psi_{\mathbf{n}}'] \langle \hat{a}_{\mathbf{j}}^\dagger \hat{a}_{\mathbf{k}} \hat{a}_{\mathbf{l}}^\dagger \hat{a}_{\mathbf{n}} \rangle$$

Two major differences appear compared to the mean flux calculation: the terms in v_3 and the phase factor $\delta_{\alpha} + 3\pi/2$ in Eq.(5) do not cancel. This makes the exact calculation very tedious. It is postponed to the appendix.

Experiments are usually performed in situations satisfying two conditions: (1) the width of the cloud after expansion is much larger than that of the trapped cloud, and (2) the mean velocity acquired during free fall is much larger than the velocity spread of the trapped cloud. The first condition means that $\omega_{\alpha} t \gg 1$ and the second one that $gt \gg \sqrt{k_B T/m}$. The latter condition also means that the mean arrival time, $t_0 = \sqrt{2H/g}$, is much larger than the time width $\sqrt{k_B T/mg^2}$ of the expanding cloud. With these approximations the scale factors become quite simple. $\tilde{x} \sim \frac{x}{\omega_x t_0}$, $\tilde{y} \sim \frac{y}{\omega_y t_0}$ and $\tilde{z} \sim \frac{H - \frac{1}{2}gt^2}{\omega_z t_0} \sim \frac{g(t_0 - t)}{\omega_z}$. In particular, the coordinate \tilde{z} is proportional to the arrival time t . This means that in experiments that

measure arrival times, the results have the same form when expressed as a function of vertical position.

In the correlation function of the flux, the above approximations also lead to $v_2 \approx \sqrt{2gH}$ and $|\sqrt{j_z} v_3/v_2| \approx \sqrt{\frac{k_B T}{\hbar \omega_z} \frac{\sigma_z}{\sqrt{2}H}} = \frac{s_z}{\sqrt{2}H}$ where s_z is the width of the cloud inside the trap and where the typical value of the occupied trap level, j_z , is $\sim \frac{k_B T}{\hbar \omega_z}$. The term containing v_3 is then very small compared to the one proportional v_2 . In Ref.[15] for instance the above ratio is $\sim 10^{-5}$. We will neglect terms containing v_3 in the following. The phase factors δ_α in Eq.(5) are also very small since $\omega_\alpha t \gg 1$ and can be neglected (see appendix VID 2).

Under all these approximations, one finds

$$G_{fl}^{(2)}(\mathbf{r}, t; \mathbf{r}', t') = \frac{v_2 v_2'}{\prod_\alpha \sqrt{(1 + \omega_\alpha^2 t^2)(1 + \omega_\alpha^2 t'^2)}} \left(\rho_{eq}(\tilde{\mathbf{r}}) \rho_{eq}(\tilde{\mathbf{r}}') + |G^{(1)}(\tilde{\mathbf{r}}, \tilde{\mathbf{r}}')|^2 - \rho_0(\tilde{\mathbf{r}}) \rho_0(\tilde{\mathbf{r}}') \right)$$

We again find the same correlation function as in the trap, rescaled by a slightly different factor compared to $G_{im.}^{(2)}$. This factor simply reflects the expansion of the cloud between the times t and t' .

The scaling laws for the harmonic potential result in a very simple expression for the correlation lengths at the detector:

$$l_\alpha^{(d)} = l^{(t)} \times \sqrt{1 + (\omega_\alpha t)^2}. \quad (12)$$

Where $l_\alpha^{(d)}$ is the correlation length along the α direction at the detector and $l_\alpha^{(t)}$ is the correlation length in the trap. If the gas is far from degeneracy $l^{(t)} = \frac{\lambda}{\sqrt{2\pi}}$, and we recover the result of Eq. 3. Close to degeneracy the correlation length is position dependent. In the case of a pulse of atoms as in Ref. [15], this formula applies along all three space axes. In addition, when making a flux measurement, one often expresses the longitudinal correlation length as a correlation time. For a pulse of atoms from a harmonic trap, with a mean velocity v at the detector, the correlation time is:

$$t^{(coh)} = \frac{l_z^{(d)}}{v} = l^{(t)} \times \frac{\omega_z}{g}. \quad (13)$$

It is independent of the propagation time as long as $\omega_z t \gg 1$.

These calculations are illustrated in the following figures. For simplicity we have used an isotropic trapping potential. As pointed out above, the normalized second-order correlation

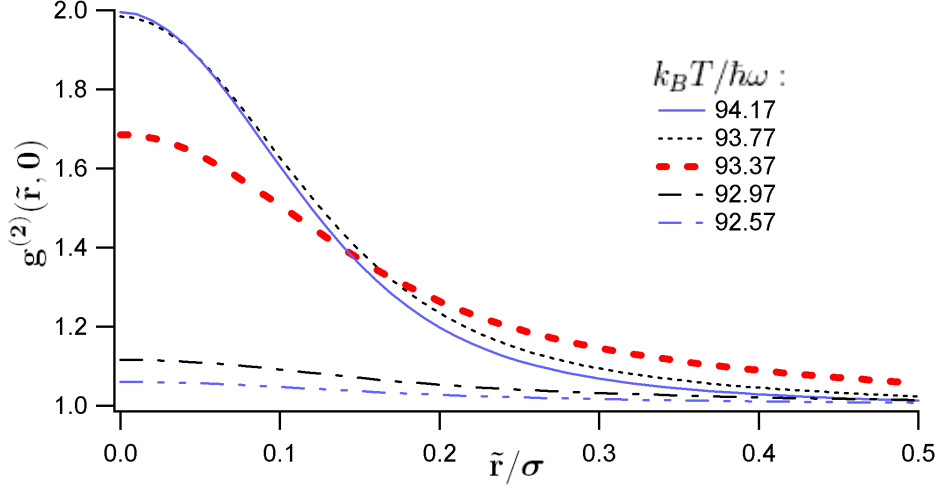


FIG. 1: (Color online) Two-body normalized correlation function at the trap center, $g^{(2)}(\tilde{r}, 0)$ for 10^6 atoms as function of the position $\tilde{r} = r/\omega t$ for various temperatures around transition temperature. The horizontal axis is labelled in units of the size of the harmonic oscillator wave function σ . The thick dashed line corresponds to the transition temperature T^* defined in Ref.[31] and is $93.37 \hbar\omega/k_B$ for 10^6 atoms. The temperature step is $0.4 \hbar\omega/k_B$. The thermal de Broglie wavelength is $\sim 0.26 \sigma$. The effect of the ground state population is clearly visible in the reduction of $g^{(2)}(0, 0)$, and in the rapid flattening out of the correlation function slightly below T^* .

functions $g_{im.}^{(2)}$ and $g_{fl.}^{(2)}$ are virtually identical with typical parameters (see appendix VIC) and we will use the shorter notation $g^{(2)}$. In Fig. 1 we show the normalized correlation function $g^{(2)}(\tilde{r}, 0)$ as a function of $\tilde{r} \sim r/\omega t$ for various temperatures in the vicinity the Bose-Einstein phase transition T^* . We use the saturation of the excited state population to define T^* [31]. This is the correlation function *at the center of the cloud*. One sees that at $T = T^*$ (the thick dashed line in the figure), the correlation function at zero distance is already significantly diminished compared to higher temperatures. The correlation length, on the other hand, is larger than $\lambda\omega t/\sqrt{2\pi}$. Also, one sees that the correlation function is almost flat for temperatures a few percent below T^* .

In many experiments of course, one does not measure the local correlation function, but the correlation function averaged over all points in the sample [15]. The effect of this averaging is shown in Fig. 2. We plot $g_m^{(2)}(\tilde{r}) = \frac{\int d\mathbf{R} G^{(2)}(\mathbf{R} + \tilde{r}\mathbf{e}, \mathbf{R})}{\int d\mathbf{R} G^{(1)}(\mathbf{R} + \tilde{r}\mathbf{e}, \mathbf{R} + \tilde{r}\mathbf{e}) G^{(1)}(\mathbf{R}, \mathbf{R})}$ where the vector \mathbf{e} is a unit vector in some direction. One sees that the amplitude of the correlation function

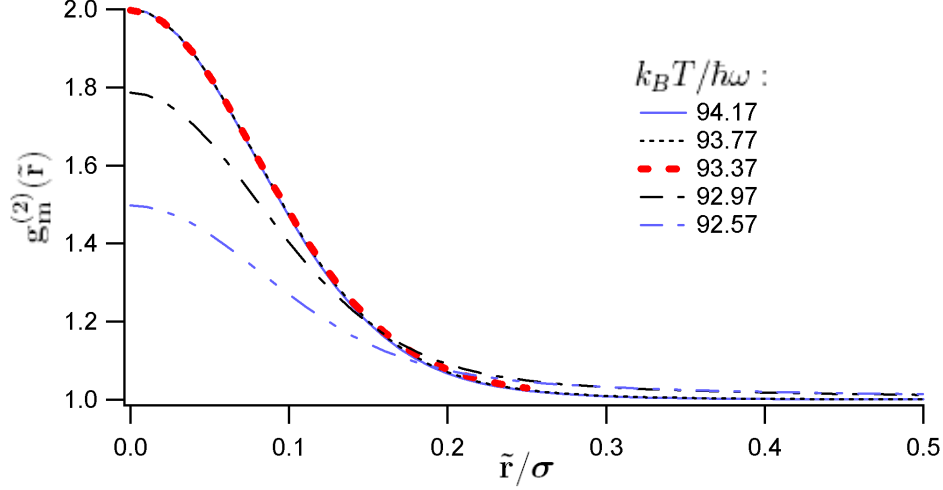


FIG. 2: (Color online) Two-body normalized correlation function $g_m^{(2)}(\tilde{r})$ for 10^6 atoms as a function of \tilde{r} . This function is an average of the two-body correlation function over the cloud. The conditions are the same conditions as for Fig.1. Unlike Fig.1, the shape is always almost Gaussian and converges more slowly to a flat correlation for low temperatures. This is because only a small region around $\tilde{r} = 0$ is fully sensitive to the quantum atomic distribution.

decreases more slowly, and that after averaging, the correlation length hardly varies as one passes T^* .

To illustrate how local the effects which distinguish Figs. 1 and 2 are, we also plot in Fig. 3 the value of $g^{(2)}(\tilde{r}, \tilde{r})$, the zero distance correlation function as a function of \tilde{r} in the vicinity of the cloud center. One sees that even below T^* , the correlator is close to 2 at a rescaled distance of a few times the harmonic oscillator length scale. We can simply interpret this effect by observing that at \tilde{r} the effective chemical potential is $\mu - V(\tilde{r})$. Away from the center, the effective chemical potential is small and this part of the cloud can be described as a Boltzmann cloud.

Before interpreting these results further, we recall some of our assumptions and their possible violation. First, we obtain Eq.(12) if we make a semi classical approximation assuming that $k_B T$ greatly exceeds the energy spacing in the trap in each dimension of space. In an anisotropic trap, this condition can be violated in one or two dimensions and then correlation length along these directions will be larger and can become infinite for a small enough temperature. Second, we have assumed a non-interacting gas throughout.

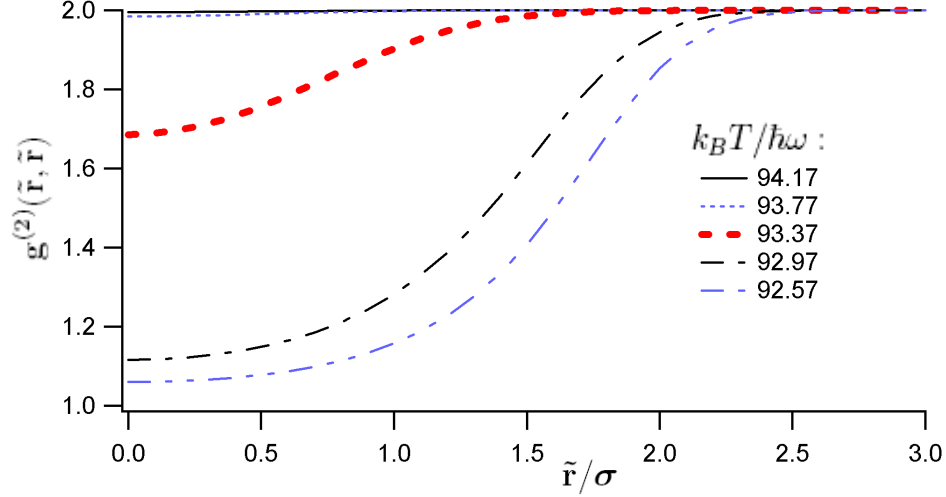


FIG. 3: (Color online) Two-body normalized correlation function $g^{(2)}(\tilde{r}, \tilde{r})$ for 10^6 atoms as function of \tilde{r} . The conditions are the same as for Fig.1. Even for $T < T^*$ the correlation goes to 2 far from the center. This is due to the finite spatial extent of the condensate. It can also be understood in terms of the chemical potential $\mu(\tilde{r})$ which, in a local density approximation, decreases as \tilde{r} increases and thus the correlation is equivalent to that of a hotter cloud.

Repulsive interactions inflate the trapped cloud, and thus reduce the length $l^{(d)}$ at the detector. We expect this to be the main effect for atomic clouds above the Bose-Einstein transition threshold, where the effects of atomic interactions are typically small. The reduction is typically a few percent. Even slightly below T^* , the condensate density is quite high, expelling the thermal atoms from the center of the trap. The effects of interactions inside the trap and during the cloud's expansion cannot be neglected. Taking them into account is then complex and beyond the scope of this paper.

III. PHYSICAL INTERPRETATIONS

The main result of this paper is that in an experiment which averages over a detector in the sense of Fig. 2, even at $T = T^*$, the correlation lengths at the detector are well approximated by:

$$l_{\alpha}^{(d)} = l^{(t)} \times \omega_{\alpha} t$$

The correlation length increases linearly with the time of flight. A simple way to understand this result is to consider the analogy with optical speckle. Increasing the time of flight corresponds to increasing the propagation distance to the observation plane in the optical analog. The speckle size, *i.e.* the correlation length, obviously increases linearly with the propagation distance. Another way to understand the time dependence is to remark that after release, the atomic cloud is free and the phase space density should be constant. Since the density decreases with time as $\prod_{\alpha}(\omega_{\alpha}t)$ and the spread of the velocity distribution is constant, the correlation volume must increase by the same factor [25].

Yet another way to look at the correlation length is to observe that, far from degeneracy, the correlation length inside the trap is the thermal de Broglie wavelength, that is, $\frac{\lambda}{\sqrt{2\pi}} = \hbar/\Delta p$ where $\Delta p = m\Delta v$ is the momentum width of the cloud. By analogy, after expansion, the correlation length is $\hbar/(\Delta p)_{loc}$, where $(\Delta p)_{loc}$ is the “local” width of the momentum distribution. As the pulse of atoms propagates, fast and slow atoms separate, so that at a given point in space the width in momentum is reduced by a factor $\frac{s_{\alpha}}{\Delta vt}$.

For a continuous beam, the formula (12) only applies in the transverse directions. In the longitudinal direction, an argument in terms of a local thermal de Broglie wavelength can be used to find the coherence length or time. If the atoms travel at velocity v without acceleration, the momentum spread and correlation length remain constant. Defining the energy width of the beam as $\Delta E = mv\Delta v$, one finds a correlation time $\lambda/v = \hbar/\Delta E$ [7]. In the presence of an acceleration such as gravity, the momentum spread of the beam decreases (the energy spread at any point ΔE is constant), which increases the correlation length. The correlation time, however, remains $\hbar/\Delta E$ [10].

The result that the coherence length of a cloud of atoms can vary with the distance of propagation, is in apparent contradiction with the results of Refs. [39, 40]. Those papers give convincing reasons, both experimental and theoretical, for why the dispersion associated with the propagation of massive particles should *not* result in an increase of the coherence length. The contradiction is resolved by noting that the Mach-Zender interferometer considered in that work is sensitive to the function $f(\mathbf{r}, t) = \int d\mathbf{R} G^{(1)}(\mathbf{R}, t; \mathbf{R} + \mathbf{r}, t)$. If the Hamiltonian commutes with the momentum operator, *i.e.* if plane waves are stationary states, one can easily demonstrate that the function f and hence its width are independent of the time t . The experiments we analyze are sensitive to the *modulus* of $G^{(1)}$ whose width will always increase with time. Thus the coherence length can depend on the interferometer as well as

the source.

The role of the acceleration of gravity in these experiments is minor. It governs the propagation time and the speed of the particles when they reach the detector. In a pulsed beam, gravity has no effect on the correlation length, although it does affect the correlation time. It also renders the rescaling of the z coordinate linear for large times so that the correlation function in position z and time have the same form. Without gravity (cancellation with a magnetic field gradient for example), a pulse of atoms would take longer to reach the detector, thereby giving the correlation length more time to dilate, and in addition they would hit the detector at a lower velocity. The correlation time would then increase with time and its order of magnitude would be $\frac{\lambda\omega t_0}{v_T} = \frac{\hbar\omega}{k_B T} t_0$ where $v_T = \sqrt{\frac{k_B T}{m}}$ is the thermal velocity and $t_0 = v_T/H$ is the time of flight to the detector.

IV. EFFECT OF FINITE DETECTOR RESOLUTION

In the preceding sections, the detector was considered ideal, *i.e.* with arbitrarily good spatial and temporal resolution. Here we will consider a model of a more realistic detector, in which we suppose that the spatial resolution in the $x-y$ plane is Gaussian. This is often the case due to smearing in pixels [13, 16] and is also approximately true in Ref. [15]. To simplify the discussion we will restrict our analysis to the case $T \gg T^*$ and use a Maxwell-Boltzmann distribution rather than Bose-Einstein distribution. In this case, each direction of space is independent and we will only consider one direction at a time in the following.

There are three different scales in the problem: the size of the cloud at the detector $s(t) \approx \sqrt{\frac{k_B T}{m}} t$, the correlation length at the detector $l^{(d)}$ and the r.m.s. width of the detector resolution function d . The definition of the resolution function is that for a density $\rho(x) = A e^{-\frac{x^2}{2s(t)^2}}$, the observed density is given by a convolution:

$$\rho_{obs}(x) = \int dx_0 \rho(x_0) \frac{e^{-\frac{1}{2}(\frac{x-x_0}{d})^2}}{\sqrt{2\pi}d} = \frac{A}{\sqrt{1 + d^2/s(t)^2}} e^{-\frac{x^2}{2[s(t)^2 + d^2]}}.$$

Similarly if $G^{(1)}(x, x') = A e^{i\phi} e^{-\frac{(x+x')^2}{2(2s)^2}} e^{-\frac{(x-x')^2}{2(l^{(d)})^2}}$ is the first order correlation function and $G_{obs}^{(1)}(x, x')$ the observed one, we have

$$|G_{obs}^{(1)}(x, x')|^2 = \int dx_0 dx'_0 |G^{(1)}(x_0, x'_0)|^2 \frac{e^{-\frac{1}{2}(\frac{x-x_0}{d})^2}}{\sqrt{2\pi}d} \frac{e^{-\frac{1}{2}(\frac{x'-x'_0}{d})^2}}{\sqrt{2\pi}d} \quad (14)$$

$$= \frac{|A|^2}{\sqrt{(1 + d^2/s^2(t))(1 + 4d^2/(l^{(d)})^2)}} e^{-\frac{(x+x')^2}{4[s(t)^2+d^2]}} e^{-\frac{(x-x')^2}{(l^{(d)})^2+4d^2}} \quad (15)$$

Consequently, with $\alpha = x, y$ and z :

- The amplitude of the normalized correlation function becomes

$$g_{obs}^{(2)}(\mathbf{0}, \mathbf{0}) = \left(\frac{G_{obs}^{(1)}(\mathbf{0}, \mathbf{0})}{\rho_{obs}(\mathbf{0})} \right)^2 = 1 + \prod_{\alpha} \sqrt{\frac{1+d_{\alpha}^2/s_{\alpha}^2(t)}{1+4d_{\alpha}^2/(l_{\alpha}^{(d)})^2}}.$$

- The observed widths of the cloud are $s_{\alpha}(t) \rightarrow \sqrt{s_{\alpha}^2(t) + d_{\alpha}^2}$.
- The observed correlation lengths are $l_{\alpha}^{(d)} \rightarrow \sqrt{(l_{\alpha}^{(d)})^2 + (2d_{\alpha})^2}$. The factor 2 can be understood as $\sqrt{2} \times \sqrt{2}$ where the first term comes from the fact that d_{α} is defined for one particle and not for a pair of particles and the second one comes from the fact that the correlation length is not defined as an r.m.s. width.

In the experiment of Ref. [15] the trapped cloud had a cigar shape. At the detector the cloud was spherical but the correlation volume was anisotropic with $l_x^{(d)} \ll d \approx l_y^{(d)}/4$. In the third (vertical) direction, the resolution width was much smaller than any other length scale. The observed contrast of the correlation function was therefore approximately, $\frac{l_x^{(d)}}{2d}$.

V. CONCLUSION

The most important conclusion of this paper is that the expansion of a non-interacting cloud from a harmonic trap in thermal equilibrium, admits a rather simple, analytical treatment of the time variation of the density and the correlation functions. In such a pulse of atoms, correlation lengths scale in the same way as the size of the density profile. The agreement with experiment indicates that the neglect of interactions is a good approximation above the BEC transition temperature. An important next step however, is to examine interaction effects so that the next generation of experiments, which will be more precise and better resolved, can be fully interpreted.

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VI. APPENDIX

A. Explicit expression of the flux correlation function

We found in section II B, the following expression for the flux operator:

$$\hat{I}(\mathbf{r}, t) = \sum_{\mathbf{j}, \mathbf{k}} \left[v_2 \psi_{\mathbf{j}}^* \psi_{\mathbf{k}} - \frac{1}{2} \left(v_3 \sqrt{k} \psi_{\mathbf{j}}^* \psi_{\mathbf{k}-\mathbf{1}_z} + v_3^* \sqrt{j} \psi_{\mathbf{j}-\mathbf{1}_z}^* \psi_{\mathbf{k}} \right) \right] \hat{a}_{\mathbf{j}}^\dagger \hat{a}_{\mathbf{k}}$$

where $\mathbf{j} - \mathbf{1}_z$ is the vector $(j_x, j_y, j_z - 1)$ and where we write $\psi = \psi(\mathbf{r}, t)$.

The second order correlation function for the flux is then,

$$\begin{aligned} \langle \hat{I}(\mathbf{r}, t) \hat{I}(\mathbf{r}', t') \rangle = & \sum_{\mathbf{j}, \mathbf{k}, \mathbf{l}, \mathbf{n}} \left[v_2 \psi_{\mathbf{j}}^* \psi_{\mathbf{k}} - \frac{1}{2} \left(v_3 \sqrt{k_z} \psi_{\mathbf{j}}^* \psi_{\mathbf{k}-\mathbf{1}_z} + v_3^* \sqrt{j_z} \psi_{\mathbf{j}-\mathbf{1}_z}^* \psi_{\mathbf{k}} \right) \right] \\ & \times \left[v_2' \psi_{\mathbf{l}}^* \psi_{\mathbf{n}}' - \frac{1}{2} \left(v_3' \sqrt{n_z} \psi_{\mathbf{l}}^* \psi_{\mathbf{n}-\mathbf{1}_z}' + v_3'^* \sqrt{l_z} \psi_{\mathbf{l}-\mathbf{1}_z}'^* \psi_{\mathbf{n}}' \right) \right] \langle \hat{a}_{\mathbf{j}}^\dagger \hat{a}_{\mathbf{k}} \hat{a}_{\mathbf{l}}^\dagger \hat{a}_{\mathbf{n}} \rangle \end{aligned}$$

Neglecting the shot-noise and ground-state contributions, this leads to

$$\langle \hat{I}(\mathbf{r}, t) \hat{I}(\mathbf{r}', t') \rangle = \langle \hat{I}(\mathbf{r}, t) \rangle \langle \hat{I}(\mathbf{r}', t') \rangle + \text{Re}(A)$$

with

$$\begin{aligned} A = \sum_{\mathbf{j}, \mathbf{l}} & [v_2 v_2' \psi_{\mathbf{j}}^* \psi_{\mathbf{j}}' \psi_{\mathbf{l}} \psi_{\mathbf{l}}'^* + \frac{1}{2} v_3 v_3' \sqrt{j_z l_z} \psi_{\mathbf{j}}^* \psi_{\mathbf{j}-\mathbf{1}_z}' \psi_{\mathbf{l}-\mathbf{1}_z} \psi_{\mathbf{l}}'^* + \frac{1}{2} v_3 v_3'^* l_z \psi_{\mathbf{j}}^* \psi_{\mathbf{j}}' \psi_{\mathbf{l}-\mathbf{1}_z} \psi_{\mathbf{l}-\mathbf{1}_z}'^* \\ & - v_2 v_3' \sqrt{j_z} \psi_{\mathbf{j}}^* \psi_{\mathbf{j}-\mathbf{1}_z}' \psi_{\mathbf{l}} \psi_{\mathbf{l}}'^* - v_2' v_3 \sqrt{l_z} \psi_{\mathbf{j}}^* \psi_{\mathbf{j}}' \psi_{\mathbf{l}-\mathbf{1}_z} \psi_{\mathbf{l}}'^*] \langle \hat{a}_{\mathbf{j}}^\dagger \hat{a}_{\mathbf{j}} \rangle \langle \hat{a}_{\mathbf{l}}^\dagger \hat{a}_{\mathbf{l}} \rangle \end{aligned}$$

We write $A = \sum_{i=1}^5 T_i$ where the T_i terms can be recast, using $\tan \delta_\alpha = 1/\omega_\alpha t$, $\tan \delta'_\alpha = 1/\omega_\alpha t'$, $\Delta_\alpha = \delta'_\alpha - \delta_\alpha$, $\sum_\alpha j_\alpha (\delta'_\alpha - \delta_\alpha) = \mathbf{j} \cdot \mathbf{\Delta}$, $\psi_{\mathbf{l}}^0 = \psi_{\mathbf{l}}^0(\tilde{\mathbf{r}})$ and $\psi_{\mathbf{l}}'^0 = \psi_{\mathbf{l}}^0(\tilde{\mathbf{r}}')$.

- $T_1 = v_2 v_2' \sum_{\mathbf{j}, \mathbf{l}} \psi_{\mathbf{j}}^* \psi_{\mathbf{j}}' \psi_{\mathbf{l}} \psi_{\mathbf{l}}'^* \langle \hat{a}_{\mathbf{j}}^\dagger \hat{a}_{\mathbf{j}} \rangle \langle \hat{a}_{\mathbf{l}}^\dagger \hat{a}_{\mathbf{l}} \rangle$

$$= \frac{v_2 v_2'}{\prod_\alpha \sqrt{(1+\omega_\alpha^2 t^2)(1+\omega_\alpha^2 t'^2)}} \sum_{\mathbf{j}, \mathbf{l}} \psi_{\mathbf{j}}^0 \psi_{\mathbf{j}}'^0 \psi_{\mathbf{l}}^0 \psi_{\mathbf{l}}'^0 e^{i \sum_\alpha (j_\alpha - l_\alpha) (\delta'_\alpha - \delta_\alpha)} \langle \hat{a}_{\mathbf{j}}^\dagger \hat{a}_{\mathbf{j}} \rangle \langle \hat{a}_{\mathbf{l}}^\dagger \hat{a}_{\mathbf{l}} \rangle$$

$$= \frac{v_2 v_2'}{\prod_\alpha \sqrt{(1+\omega_\alpha^2 t^2)(1+\omega_\alpha^2 t'^2)}} \left| \sum_{\mathbf{j}} \psi_{\mathbf{j}}^0 \psi_{\mathbf{j}}'^0 e^{i \mathbf{j} \cdot \mathbf{\Delta}} \langle \hat{a}_{\mathbf{j}}^\dagger \hat{a}_{\mathbf{j}} \rangle \right|^2$$
- $T_2 = \frac{1}{2} v_3 v_3' \sum_{\mathbf{j}, \mathbf{l}} \sqrt{j_z l_z} \psi_{\mathbf{j}}^* \psi_{\mathbf{j}-\mathbf{1}_z}' \psi_{\mathbf{l}-\mathbf{1}_z} \psi_{\mathbf{l}}'^* \langle \hat{a}_{\mathbf{j}}^\dagger \hat{a}_{\mathbf{j}} \rangle \langle \hat{a}_{\mathbf{l}}^\dagger \hat{a}_{\mathbf{l}} \rangle$

$$= -\frac{1}{2} \frac{|v_3 v_3'|}{\prod_\alpha \sqrt{(1+\omega_\alpha^2 t^2)(1+\omega_\alpha^2 t'^2)}} \left(\sum_{\mathbf{j}} \sqrt{j_z} \psi_{\mathbf{j}}^0 \psi_{\mathbf{j}-\mathbf{1}_z}'^0 e^{i \mathbf{j} \cdot \mathbf{\Delta}} \langle \hat{a}_{\mathbf{j}}^\dagger \hat{a}_{\mathbf{j}} \rangle \right) \left(\sum_{\mathbf{l}} \sqrt{l_z} \psi_{\mathbf{l}-\mathbf{1}_z}^0 \psi_{\mathbf{l}}'^0 e^{-i \mathbf{l} \cdot \mathbf{\Delta}} \langle \hat{a}_{\mathbf{l}}^\dagger \hat{a}_{\mathbf{l}} \rangle \right)$$

- $$T_3 = \frac{1}{2} v_3 v_3'^* \sum_{\mathbf{j}, \mathbf{l}} l_z \psi_{\mathbf{j}}^* \psi_{\mathbf{j}}' \psi_{\mathbf{l}-\mathbf{1}_z} \psi_{\mathbf{l}-\mathbf{1}_z}'^* \langle \hat{a}_{\mathbf{j}}^\dagger \hat{a}_{\mathbf{j}} \rangle \langle \hat{a}_{\mathbf{l}}^\dagger \hat{a}_{\mathbf{l}} \rangle$$

$$= \frac{1}{2} \frac{|v_3 v_3'|}{\prod_{\alpha} \sqrt{(1+\omega_{\alpha}^2 t^2)(1+\omega_{\alpha}^2 t'^2)}} \left(\sum_{\mathbf{j}} \psi_{\mathbf{j}}^0 \psi_{\mathbf{j}}'^0 e^{i\mathbf{j} \cdot \Delta} \langle \hat{a}_{\mathbf{j}}^\dagger \hat{a}_{\mathbf{j}} \rangle \right) \left(\sum_{\mathbf{l}} l_z \psi_{\mathbf{l}-\mathbf{1}_z}^0 \psi_{\mathbf{l}-\mathbf{1}_z}'^0 e^{-i\mathbf{l} \cdot \Delta} \langle \hat{a}_{\mathbf{l}}^\dagger \hat{a}_{\mathbf{l}} \rangle \right)$$
- $$T_4 = -v_2 v_3' \sum_{\mathbf{j}, \mathbf{l}} \sqrt{j_z} \psi_{\mathbf{j}}^* \psi_{\mathbf{j}-\mathbf{1}_z}' \psi_{\mathbf{l}} \psi_{\mathbf{l}}'^* \langle \hat{a}_{\mathbf{j}}^\dagger \hat{a}_{\mathbf{j}} \rangle \langle \hat{a}_{\mathbf{l}}^\dagger \hat{a}_{\mathbf{l}} \rangle$$

$$= -i \frac{v_2 |v_3'|}{\prod_{\alpha} \sqrt{(1+\omega_{\alpha}^2 t^2)(1+\omega_{\alpha}^2 t'^2)}} \left(\sum_{\mathbf{j}} \sqrt{j_z} \psi_{\mathbf{j}}^0 \psi_{\mathbf{j}-\mathbf{1}_z}'^0 e^{i\mathbf{j} \cdot \Delta} \langle \hat{a}_{\mathbf{j}}^\dagger \hat{a}_{\mathbf{j}} \rangle \right) \left(\sum_{\mathbf{l}} \psi_{\mathbf{l}}^0 \psi_{\mathbf{l}}'^0 e^{-i\mathbf{l} \cdot \Delta} \langle \hat{a}_{\mathbf{l}}^\dagger \hat{a}_{\mathbf{l}} \rangle \right)$$
- $$T_5 = -v_2' v_3 \sum_{\mathbf{j}, \mathbf{l}} \sqrt{l_z} \psi_{\mathbf{j}}^* \psi_{\mathbf{j}}' \psi_{\mathbf{l}-\mathbf{1}_z} \psi_{\mathbf{l}}'^* \langle \hat{a}_{\mathbf{j}}^\dagger \hat{a}_{\mathbf{j}} \rangle \langle \hat{a}_{\mathbf{l}}^\dagger \hat{a}_{\mathbf{l}} \rangle$$

$$= -i \frac{v_2' |v_3|}{\prod_{\alpha} \sqrt{(1+\omega_{\alpha}^2 t^2)(1+\omega_{\alpha}^2 t'^2)}} \left(\sum_{\mathbf{j}} \psi_{\mathbf{j}}^0 \psi_{\mathbf{j}}'^0 e^{i\mathbf{j} \cdot \Delta} \langle \hat{a}_{\mathbf{j}}^\dagger \hat{a}_{\mathbf{j}} \rangle \right) \left(\sum_{\mathbf{l}} \sqrt{l_z} \psi_{\mathbf{l}-\mathbf{1}_z}^0 \psi_{\mathbf{l}}'^0 e^{-i\mathbf{l} \cdot \Delta} \langle \hat{a}_{\mathbf{l}}^\dagger \hat{a}_{\mathbf{l}} \rangle \right)$$

The term T_1 is a real number which is not the case for T_2, T_3, T_4 and T_5 .

B. Calculation for harmonic oscillator stationary states

All the above terms can be calculated analytically. All the series are identical in the direction x and y . We are then left with the calculation of three series in only one direction:

- $$\sum_{n=0}^{\infty} \sqrt{n} \psi_{n-1}^0(\tilde{z}) \psi_n^0(\tilde{z}') e^{-nu}$$
- $$\sum_{n=0}^{\infty} \sqrt{n} \psi_n^0(\tilde{z}) \psi_{n-1}^0(\tilde{z}') e^{-nu}$$
- $$\sum_{n=0}^{\infty} n \psi_{n-1}^0(\tilde{z}) \psi_{n-1}^0(\tilde{z}') e^{-nu}$$

The function $g_u(\tilde{z}, \tilde{z}') = \sum_{n=0}^{\infty} \psi_n^0(\tilde{z}) \psi_n^0(\tilde{z}') e^{-nu}$ is known [17, 29] and its expression is $g_u(\tilde{z}, \tilde{z}') = \frac{1}{\sigma \sqrt{\pi(1-e^{-2u})}} \exp[-\tanh(\frac{u}{2}) \left(\frac{\tilde{z}+\tilde{z}'}{2\sigma}\right)^2 - \coth(\frac{u}{2}) \left(\frac{\tilde{z}-\tilde{z}'}{2\sigma}\right)^2]$.

Using $\tilde{z} \psi_n^0(\tilde{z}) = \frac{\sigma}{\sqrt{2}} \langle \tilde{z} | \hat{a} + \hat{a}^\dagger | \psi_n^0 \rangle = \frac{\sigma}{\sqrt{2}} [\sqrt{n} \psi_{n-1}^0(\tilde{z}) + \sqrt{n+1} \psi_{n+1}^0(\tilde{z})]$, one finds

$$\tilde{z} g_u(\tilde{z}, \tilde{z}') = \frac{\sigma}{\sqrt{2}} [\sum \sqrt{n} \psi_{n-1}^0(\tilde{z}) \psi_n^0(\tilde{z}') e^{-nu} + e^u \sum \sqrt{n} \psi_n^0(\tilde{z}) \psi_{n-1}^0(\tilde{z}') e^{-nu}].$$

It follows easily that

- $$\sum_{n=0}^{\infty} \sqrt{n} \psi_{n-1}^0(\tilde{z}) \psi_n^0(\tilde{z}') e^{-nu} = \frac{\sqrt{2}}{\sigma} \frac{\tilde{z} - e^u \tilde{z}'}{1 - e^{2u}} g_u(\tilde{z}, \tilde{z}')$$
- $$\sum_{n=0}^{\infty} \sqrt{n} \psi_n^0(\tilde{z}) \psi_{n-1}^0(\tilde{z}') e^{-nu} = \frac{\sqrt{2}}{\sigma} \frac{\tilde{z}' - e^u \tilde{z}}{1 - e^{2u}} g_u(\tilde{z}, \tilde{z}')$$

Moreover, $\sum_{n=0}^{\infty} n \psi_{n-1}^0(\tilde{z}) \psi_{n-1}^0(\tilde{z}') e^{-nu} = e^{-u} [g_u(\tilde{z}, \tilde{z}') - \partial_u g_u(\tilde{z}, \tilde{z}')]$. Then,

- $$\sum_{n=0}^{\infty} n \psi_{n-1}^0(\tilde{z}) \psi_{n-1}^0(\tilde{z}') e^{-nu} = \left[\frac{1}{1-e^{-2u}} + \frac{1}{2} \left(\frac{\tilde{z}+\tilde{z}'}{2\sigma \cosh \frac{u}{2}} \right)^2 - \frac{1}{2} \left(\frac{\tilde{z}-\tilde{z}'}{2\sigma \sinh \frac{u}{2}} \right)^2 \right] e^{-u} g_u(\tilde{z}, \tilde{z}')$$

C. Explicit expression of the flux correlation function-Part II

We define $G_B^{(1)}(\mathbf{r}, \mathbf{r}', \mathbf{u}) = \sum_{\mathbf{n}} \psi_{\mathbf{n}}^0(\mathbf{r}) \psi_{\mathbf{n}}^0(\mathbf{r}') e^{-\mathbf{n} \cdot \mathbf{u}}$. This function, the 3D equivalent of the function g_u , is connected to the one-body correlation function by $G^{(1)}(\mathbf{r}, \mathbf{r}') = \sum_{l=1}^{\infty} e^{\beta l \tilde{\mu}} G_B^{(1)}(\mathbf{r}, \mathbf{r}', l\boldsymbol{\tau})$ with $\tau_{\alpha} = \beta \hbar \omega_{\alpha}$.

Then,

- $T_1 = \frac{v_2 v'_2}{\prod_{\alpha} \sqrt{(1+\omega_{\alpha}^2 t^2)(1+\omega_{\alpha}^2 t'^2)}} \left| \sum_l e^{\beta l \tilde{\mu}} G_B^{(1)}(\tilde{\mathbf{r}}, \tilde{\mathbf{r}}', l\boldsymbol{\tau} - i\boldsymbol{\Delta}) \right|^2$
- $T_2 = -\frac{1}{2} \frac{|v_3 v'_3|}{\prod_{\alpha} \sqrt{(1+\omega_{\alpha}^2 t^2)(1+\omega_{\alpha}^2 t'^2)}} \left(\sum_l e^{\beta l \tilde{\mu}} \frac{\sqrt{2}}{\sigma} \frac{\tilde{z} - e^{l\tau_z - i\Delta_z} \tilde{z}'}{1 - e^{2(l\tau_z - i\Delta_z)}} G_B^{(1)}(\tilde{\mathbf{r}}, \tilde{\mathbf{r}}', l\boldsymbol{\tau} - i\boldsymbol{\Delta}) \right)$
 $\times \left(\sum_k e^{\beta k \tilde{\mu}} \frac{\sqrt{2}}{\sigma} \frac{\tilde{z}' - e^{k\tau_z + i\Delta_z} \tilde{z}}{1 - e^{2(k\tau_z + i\Delta_z)}} G_B^{(1)}(\tilde{\mathbf{r}}, \tilde{\mathbf{r}}', k\boldsymbol{\tau} + i\boldsymbol{\Delta}) \right)$
- $T_3 = \frac{1}{2} \frac{|v_3 v'_3|}{\prod_{\alpha} \sqrt{(1+\omega_{\alpha}^2 t^2)(1+\omega_{\alpha}^2 t'^2)}} \left(\sum_l e^{\beta l \tilde{\mu}} G_B^{(1)}(\tilde{\mathbf{r}}, \tilde{\mathbf{r}}', l\boldsymbol{\tau} - i\boldsymbol{\Delta}) \right)$
 $\times \left(\sum_k e^{\beta k \tilde{\mu}} \left[\frac{1}{1 - e^{2(k\tau_z + i\Delta_z)}} + \frac{1}{2} \left(\frac{\tilde{z} + \tilde{z}'}{2\sigma \cosh \frac{k\tau_z + i\Delta_z}{2}} \right)^2 - \frac{1}{2} \left(\frac{\tilde{z} - \tilde{z}'}{2\sigma \sinh \frac{k\tau_z + i\Delta_z}{2}} \right)^2 \right] e^{-(k\tau_z + i\Delta_z)} G_B^{(1)}(\tilde{\mathbf{r}}, \tilde{\mathbf{r}}', k\boldsymbol{\tau} + i\boldsymbol{\Delta}) \right)$
- $T_4 = -i \frac{v_2 |v'_3|}{\prod_{\alpha} \sqrt{(1+\omega_{\alpha}^2 t^2)(1+\omega_{\alpha}^2 t'^2)}} \left(\sum_l e^{\beta l \tilde{\mu}} \frac{\sqrt{2}}{\sigma} \frac{\tilde{z} - e^{l\tau_z - i\Delta_z} \tilde{z}'}{1 - e^{2(l\tau_z - i\Delta_z)}} G_B^{(1)}(\tilde{\mathbf{r}}, \tilde{\mathbf{r}}', l\boldsymbol{\tau} - i\boldsymbol{\Delta}) \right)$
 $\times \left(\sum_k e^{\beta k \tilde{\mu}} G_B^{(1)}(\tilde{\mathbf{r}}, \tilde{\mathbf{r}}', k\boldsymbol{\tau} + i\boldsymbol{\Delta}) \right)$
- $T_5 = -i \frac{|v_3| v'_2}{\prod_{\alpha} \sqrt{(1+\omega_{\alpha}^2 t^2)(1+\omega_{\alpha}^2 t'^2)}} \left(\sum_l e^{\beta l \tilde{\mu}} G_B^{(1)}(\tilde{\mathbf{r}}, \tilde{\mathbf{r}}', l\boldsymbol{\tau} - i\boldsymbol{\Delta}) \right)$
 $\times \left(\sum_k e^{\beta k \tilde{\mu}} \frac{\sqrt{2}}{\sigma} \frac{\tilde{z}' - e^{k\tau_z + i\Delta_z} \tilde{z}}{1 - e^{2(k\tau_z + i\Delta_z)}} G_B^{(1)}(\tilde{\mathbf{r}}, \tilde{\mathbf{r}}', k\boldsymbol{\tau} + i\boldsymbol{\Delta}) \right)$

The dominant term is T_1 and is the one used in section II B 5.

D. Contribution of neglected terms in the correlation of the flux

Here we evaluate the neglected the terms T_2 to T_5 and the shot-noise contribution. They will be evaluated in the case of clouds far above BEC threshold. Under this assumption, all the functions are separable in the variables x, y and t and the summation over the index l in the previous equations reduces to the single term $l = 1$.

1. Shot-noise contribution

Using the above analysis one can show that the main term is still proportional to $v_2 v'_2$. The additional term is then,

$$\frac{v_2 v'_2}{\prod_{\alpha} \sqrt{(1 + \omega_{\alpha}^2 t^2)(1 + \omega_{\alpha}^2 t'^2)}} e^{\beta \tilde{\mu}} G_B^{(1)}(\tilde{\mathbf{r}}, \tilde{\mathbf{r}}', \boldsymbol{\tau} - i\boldsymbol{\Delta}) G_B^{(1)}(\tilde{\mathbf{r}}, \tilde{\mathbf{r}}', i\boldsymbol{\Delta})$$

For $t = t'$, $\boldsymbol{\Delta} = \mathbf{0}$ and $G_B^{(1)}(\tilde{\mathbf{r}}, \tilde{\mathbf{r}}', \mathbf{0}) = \delta(\tilde{\mathbf{r}} - \tilde{\mathbf{r}}')$. The shot-noise term is then

$$\frac{v_2^2}{\prod_{\alpha} (1 + \omega_{\alpha}^2 t^2)} \rho_{eq}(\tilde{\mathbf{r}}) \delta(\tilde{\mathbf{r}} - \tilde{\mathbf{r}}')$$

As expected, this term corresponds also to the one at equilibrium with rescaled coordinates.

2. $T_2 - T_5$ contribution

We have $G_{fl}^{(2)}(\mathbf{r}, t; \mathbf{r}', t') = \langle \hat{I}(\mathbf{r}, t) \hat{I}(\mathbf{r}', t') \rangle = \langle \hat{I}(\mathbf{r}, t) \rangle \langle \hat{I}(\mathbf{r}', t') \rangle + \text{Re}(A)$ where $A = \sum_{i=1}^5 T_i$

- Case $t = t'$.

- $\boldsymbol{\Delta} = \mathbf{0}$,
- then $T_1 = \frac{v_2 v'_2}{\prod_{\alpha} \sqrt{(1 + \omega_{\alpha}^2 t^2)(1 + \omega_{\alpha}^2 t'^2)}} |G^{(1)}(\tilde{\mathbf{r}}, \tilde{\mathbf{r}}')|^2$, T_2 and T_3 are real number and $\text{Re}(T_4) = \text{Re}(T_5) = 0$.
- One finds, to leading orders,
 $g^{(2)}(0, 0, t; 0, 0, t) - 2 \approx \frac{1}{8} \left(\frac{s_z}{H} \right)^2 (1 - 2 \frac{t-t_0}{t_0}) (1 - \frac{\tau_z^2}{6})$ where s_z is the initial size of the cloud in the vertical direction and $t_0 = \sqrt{2H/g}$.
- The deviation from 2 is extremely small in the experimental conditions of [15] ($\sim 10^{-11}$) but shows that the bunching is strictly speaking not 2 at the center. This behavior is expected for any flux correlation function of dispersive waves [41].
- The correlation lengths at the detector are not modified by the additional terms.

- Case $t \neq t'$.

– The correlation function can be written as

$$g^{(2)}(0, 0, t; 0, 0, t') = 1 + \frac{|G_B^{(1)}(\tilde{\mathbf{r}}, \tilde{\mathbf{r}}', \boldsymbol{\tau} + i\boldsymbol{\Delta})|^2}{G_B^{(1)}(\tilde{\mathbf{r}}, \tilde{\mathbf{r}}, \boldsymbol{\tau})G_B^{(1)}(\tilde{\mathbf{r}}', \tilde{\mathbf{r}}', \boldsymbol{\tau})} [1 + \epsilon].$$

– where $\frac{|G_B^{(1)}(\tilde{\mathbf{r}}, \tilde{\mathbf{r}}', \boldsymbol{\tau} + i\boldsymbol{\Delta})|^2}{G_B^{(1)}(\tilde{\mathbf{r}}, \tilde{\mathbf{r}}, \boldsymbol{\tau})G_B^{(1)}(\tilde{\mathbf{r}}', \tilde{\mathbf{r}}', \boldsymbol{\tau})} \approx e^{-\left(\frac{t-t'}{t(\text{coh})}\right)^2 (1 - \frac{\tau_z^2}{6}) [1 - (\frac{t+t'-2t_0}{t_0})]}$ and

$$\epsilon \approx \frac{1}{8} \left(\frac{w_z}{H}\right)^2 [1 - (\frac{t+t'-2t_0}{t_0})] (1 - \frac{\tau_z^2}{6}) - \frac{3}{2(\omega_z t_0 \tau_z)^2} \left(\frac{t-t'}{t_0}\right)^2 (1 + \frac{\tau_z}{3}).$$

We have neglected terms in τ_z , $(t-t_0)^3$, $(t'-t_0)^3$, $(t-t_0)^2(t'-t_0)$, $(t-t_0)(t'-t_0)^2$ and higher orders.

– The value of ϵ is extremely small ($\sim 10^{-10}$) using Ref.[15]. The deviation from $e^{-\left(\frac{t-t'}{t(\text{coh})}\right)^2}$ is mainly due to the mean time $(t+t')/2$ contribution and changes the value of the correlation time in the wings of the time-of-flight by $\sim 3\%$. The effect of the phase factor $\boldsymbol{\Delta}$ is negligible.

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